QUADRATIC EQUATIONS

Remember solutions are the 'roots' of the equation they are also where the graph crosses the x axis

3 WAYS OF SOLVING

① Factorising

by putting equation into 2 brackets that multiply together

eg $x^2 + x - 12 = 0$ ← remember equation must always $= 0$

$(x-3)(x+4) = 0$

$x = 3$ or -4

more difficult factorising

$3x^2 + 13x + 4$

① multiply 3 and 4 together to give 12

② find 2 numbers that $\times$ to $= 12$ → $1 \times 12 = 12$
$+$ to $= 13$ → $1 + 12 = 13$

③ substitute $1 \& 12$ into equation into place of 13

$3x^2 + 1x + 12x + 4$

④ now factorise in pairs

$x(3x+1) + 4(3x+1)$

⑤ $(3x+1)$ is common so $3x^2 + 13x + 4 = (3x+1)(x+4)$

② Formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for quadratic equation

$ax^2 + bx + c = 0$ ← must $= 0$

eg. $2x^2 + 6x - 3 = 0 \Rightarrow \dfrac{-6 \pm \sqrt{6^2 - (4 \times 2 \times -3)}}{4}$

note be careful of your - signs!

$\Rightarrow \dfrac{-6 \pm \sqrt{36 + 24}}{4} \Rightarrow 0.436$ or -3.44

possible number of solutions

If $b^2 - 4ac = $ +ve number $=$ 2 solutions

$b^2 - 4ac = 0$ $=$ 1 solution

$b^2 - 4ac = $ -ve number $=$ no solutions

③ Completing the Square

looking at the equation

$x^2 + 6x$

completing the square this becomes $(x+3)^2 - 9$

Method

① look at constant infront of x & halve it
in this case $\frac{1}{2}$ of $6 = 3$

② put this into a bracket with x & square it $\Rightarrow (x+3)^2$

③ now expand $(x+3)^2 \Rightarrow x^2+6x+9$

original equation (x^2+6x)

surplus to equation so it is minused ($+9$)

$\Rightarrow (x+3)^2 - 9$

eg 2,

$x^2 - 6x + 7 = 0$

$(x-3)^2 - 9 + 7 = 0 \quad \Rightarrow (x-3)^2 - 2 = 0$

Solving using completing the square

$x^2 - 4x - 3 = 0$

$(x-2)^2 - 4 - 3 = 0 \quad \Rightarrow (x-2)^2 - 7 = 0$ ← now solve

$(x-2)^2 = 7$

$x - 2 = \pm\sqrt{7}$ ← remember $\pm\sqrt{\ }$

$x = 2 \pm \sqrt{7}$ ← answer in surd form

$x = 4.64$ or -0.646

Finding the turning point using completing the square

When ~~the~~ a quadratic equation is put into the form

$(x+a)^2 + b$

a represents the $-x$ coordinate
b represents the y coordinate
of the turning point $(-a, b)$

turning point being where the curve turns

for +ve x^2 it's a minimum point

for -ve x^2 it's a maximum point