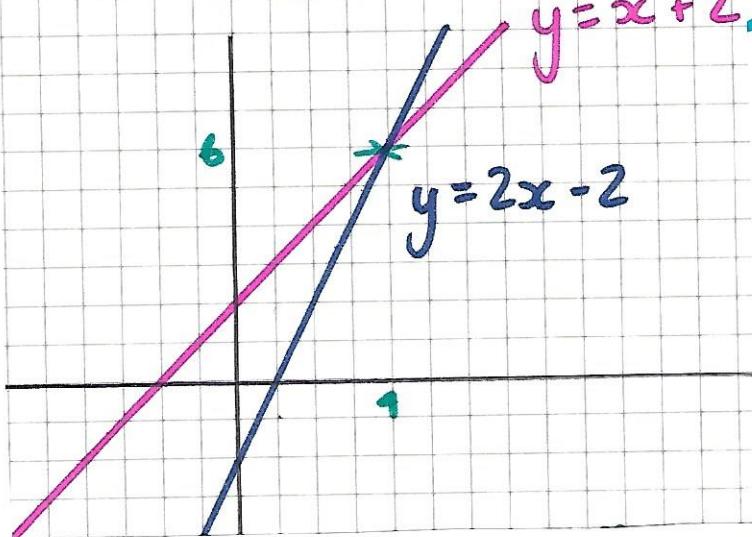


# SIMULTANEOUS EQUATIONS

This is when we have 2 equations with 2 different variables but with the same solution for both.

They can be solved graphically



The 2 Equations

$$y = x + 2$$

$$y = 2x - 2$$

meet at coordinate (4, 6)

So the solution is  $x = 4$   
 $y = 6$

Without a Graph they can be solved with either

**SUBSTITUTION METHOD**

Where we substitute the value of the one unknown into the second equation

$$\begin{aligned} \textcircled{1} \quad & y = x + 1 \\ \textcircled{2} \quad & y + 3x = 21 \end{aligned}$$

substitute  $y = x + 1$   
into 2nd equation

$$(x+1) + 3x = 21$$

Now simplify & solve

$$4x + 1 = 21$$

$$4x = 20 \Rightarrow x = 5$$

Now substitute  $x = 5$  back into  $\textcircled{1}$

$$y = 5 + 1 \quad \therefore y = 6$$

$$\begin{aligned} y &= 2x - 3 \\ 2y + 3x &= \end{aligned}$$

$y = 2x - 3$   
 $2y = 4x - 6$   
sub into  $\textcircled{1}$

$$\begin{aligned} \therefore x - 6 + 3x &= 15 \\ 7x - 6 &= 15 \end{aligned}$$

Now simplify & solve

$$7x = 21 \Rightarrow x = 3$$

now substitute  $x = 3$  back into  $\textcircled{1}$

$$\begin{aligned} y &= (2 \times 3) - 3 \\ y &= 6 - 3 \quad \underline{y = 3} \end{aligned}$$

**ELIMINATION METHOD**

By adding or subtracting one equation from the other we eliminate one unknown

$$\begin{aligned} \textcircled{1} \quad & 2x + y = 7 \\ \textcircled{2} \quad & x - y = 2 \end{aligned}$$

add  $\textcircled{1}$  to  $\textcircled{2}$   
to eliminate  $y$

$$3x = 9 \quad \leftarrow \text{Now solve}$$

$x = 3$  Finally substitute back  
into  $\textcircled{1}$  to find  $y$

$$\therefore \begin{aligned} 6 + y &= 7 \\ y &= 1 \end{aligned} \quad \text{to solve}$$

$$\begin{aligned} \textcircled{1} \quad & 5x + 2y = 8 \\ \textcircled{2} \quad & 2x + y = 3 \\ \textcircled{3} \quad & 4x + 2y = 6 \end{aligned}$$

Multiply the 2nd equation by 2  
to get  $2y$   
Now take  $\textcircled{3}$  from  $\textcircled{2}$   
to eliminate  $y$

$$x = 2$$

$$\begin{aligned} (2 \times 2) + y &= 3 \\ 4 + y &= 3 \\ y &= -1 \end{aligned}$$

Finally substitute  
 $x = 2$  back into  $\textcircled{2}$   
to find  $y$

# SIMULTANEOUS EQUATIONS

## One linear & one non-linear

$$\begin{aligned} \text{Eq 1: } y &= 2x + 3 & (1) \\ \text{Eq 2: } y &= x^2 & (2) \end{aligned}$$

- Substitute (2) into (1)

$$x^2 = 2x + 3 \quad -\text{ now rearrange to get to } = 0$$

$$x^2 - 2x - 3 = 0$$

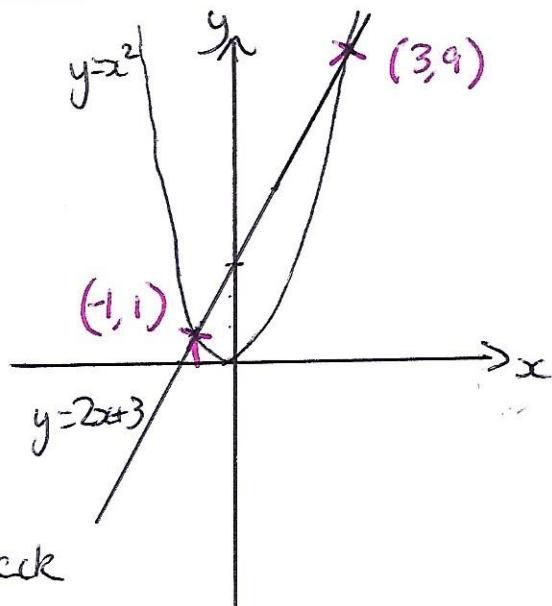
$$(x-3)(x+1) = 0 \quad \begin{matrix} \text{factorise} \\ \& \text{& solve} \end{matrix}$$

$$x = 3 \text{ or } x = -1$$

- Now substitute your 2 values of  $x$  back into equation (1) to get 2 values of  $y$

$$\begin{aligned} x = 3 & \quad y = 6 + 3 = 9 & x = -1 & \quad y = -2 + 3 = 1 \end{aligned}$$

$$\text{So } x = 3 \text{ & } y = 9 \quad \text{OR} \quad x = -1 \text{ & } y = 1$$



## When an equation of a circle is involved

$$\begin{aligned} \text{Eq 1: } x + 2y = 10 & \quad (1) \\ \boxed{x^2 + y^2 = 25} & \quad (2) \end{aligned}$$

equation of a circle  
 $x^2 + y^2 = r^2$  where  
 $r$  = radius

- In equation (1) make  $x$  the subject

$$x = 10 - 2y$$

$$-\text{ Now square it } x^2 = (10 - 2y)^2 \Rightarrow \boxed{100 - 40y + 4y^2}$$

- Substitute this into equation (2)

$$\begin{aligned} 100 - 40y + 4y^2 + y^2 &= 25 \\ 5y^2 - 40y + 75 &= 0 \quad \leftarrow \begin{matrix} \text{simplify} \\ \text{to make } = 0 \end{matrix} \\ \div 5 \quad y^2 - 8y + 15 &= 0 \\ (y-3)(y-5) &= 0 \end{aligned}$$

$$\text{So } y = 3 \text{ or } y = 5$$

Now substitute back into Eq 1  
 to get values for  $x$

$$\text{when } y = 3 \quad x = 4 \quad y = 5 \quad x = 0$$

