Recurring Decimals

All fractions can be written as decimals which either **terminate** or produce a set of **recurring** digits

Dot notation shows which digits recur

$0.\dot{4}\dot{7} = 0.474747....$ $0.\dot{1}5\dot{3} = 0.153153153....$

To express a recurring decimal as a fraction we use algebra

$x = 0.\dot{3}$

ie, $x = 0.33333.....$

$\times 10$ $10x = 3.33333$ $\times 10$

① multiply both sides by 10
② take x from $10x$ to eliminate the recurring 3
③ now ÷ by 9

$10x - x \rightarrow 9x = 3$

$x = \frac{3}{9} = \frac{1}{3}$

$x = 0.\dot{4}\dot{2}$

ie, $x = 0.424242.....$

$\times 100$ $100x = 42.424242$ $\times 100$

① this time × by 100 to keep the 42 in line
② now subtract $100x - x$

$99x = 42$

$x = \frac{42}{99} \rightarrow \frac{14}{35}$

$x = 0.1\dot{2}\dot{3}$ $x = 0.12323232$

$\times 100$ $100x = 12.32323232$ $\times 100$

$100x - x \rightarrow 99x = 12.2$

$x = \frac{12.2}{99} \rightarrow \frac{122}{990} \rightarrow \frac{61}{495}$